


TITLE	A mathematical analysis of the drying of faeces in a urine diversion toilet – sensitivity analysis
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Short CV for Introduction Purposes (100 words max)	<p>Graduated from University of Natal with BSc Eng (Che) in 1970.</p> <p>Between 1990 and 2005 I have worked as a researcher and project leader for the various WRC projects in membrane treatment, computational fluid dynamics and water pinch analysis.</p>
Photograph attached (jpg)	

INTRODUCTION

The South African Government Strategic Framework for Water Services (Department of Water Affairs and Forestry, 2003) sets as one of its goals providing access for all South Africans to an appropriate, effective and sustainable sanitation service. The *Ventilated Improved Pit Latrine* (VIP) is suggested as a model for the minimum basic level of service. Local government institutions such as municipalities have the responsibility of setting up the water and sanitation systems, and at least the basic level of service must be provided irrespective of the ability of communities to pay for services.

Pit latrines are supposed to be *on-site* sanitation systems, (i.e. the full treatment process is supposed to occur within the bounds of the property with the preferred servicing technique being relocation of the pit once full). However, in South Africa the VIP has developed into having immovable brick and mortar superstructures. This has stemmed primarily from using *status* and *dignity* as selling points for sanitation.

This implies that the pits need to be evacuated mechanically when full. The service area of eThekweni Municipality, the local authority servicing the Durban region, has very steep terrain and in the underdeveloped areas access for heavy vehicles is very restricted, if not impossible.

Currently the cost of emptying a VIP mechanically ranges between R500 and R1 500 per emptying (US\$70 to US\$200), which on average should occur every 4 years. This is generally beyond the affordability of the target population, and although there is a drive within the country to provide *free basic services*, it is still considered prudent to provide a sanitation service that is affordable to the target consumer (Department of Water Affairs and Forestry, 2002). Further, hydro-geological conditions preclude the use of pit latrines in many of the eThekweni Municipality service areas, due to low cover on bedrock, clay and high water tables. In Umzinyathi one of the suburbs of Durban, the initial hydro- geological survey revealed that only about 40 % of 5 500 sites were suitable for VIP development. VIPs in general are not a solution for sanitation in the eThekweni Municipality service area.

One of the striking features about the use of vacuum tankers to empty pit latrines is the sheer volume of water that needs to be transported by a very inefficient mode, in order to remove relatively small quantities of solid material. The logical progression from this realisation is to desiccate the material. This has a number of advantages; apart from reducing the volume, odour is reduced and the sludge can be worked with hand tools. These features present the potential for *owner servicing*, with the concomitant affordability and sustainability advantages. Some of the *ecological sanitation* systems (Esrey *et al.*, 1998) have been using desiccation for some time, however none of the applications were carried out in conditions similar to those in Durban.

Due to the urgency and scale of the roll-out of the provision of sanitation, the eThekweni Municipality found itself in the position where it has to make a multi-billion Rand decision between a technology that was not likely to work (VIPs) and one that could work, but essentially in untried circumstances. Consequently a *belt and braces* approach was adopted where the municipality has instituted a *double vault urine diversion* system. The provision also entails extensive marketing and health and hygiene education.

With a double vault latrine, one vault is used at a time. While it is filling, the contents of the idle vault are allowed to dry, hopefully to the point where they become safe and not unpleasant to handle. The vault can then be emptied in preparation for the next cycle of use. It needs to be noted that at this stage eThekweni Municipality is not promoting the full implementation of *eco-sanitation*, (i.e. the food-sludge production - soil conditioner - food cycle), but has merely borrowed some of the desiccating technology in order to solve some

sanitation problems. The sanitation system could easily be adapted to an *ecosan* system, however at the moment the priority is to develop an accepted, sustainable and hygienic sanitation system.

As the technology is untried in the application there is considerable scope for research. One of the urgent research items is the need to understand the drying mechanisms. This not only affects the design of the system but also the education of users, and could become critical in the management of abusive use.

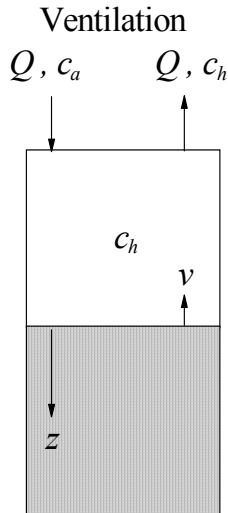


Figure 1: A double vault urine diversion latrine installed in an underdeveloped residential area

There are two obvious phases which need to be considered in analysing the drying problem, 1) while the vault is in use and gradually filling, and 2) when it is idle and its contents are undisturbed. This paper presents an analysis of the filling phase only, in which the filling itself is the dominant characteristic. During the idle phase, the physical properties of the contents are the main determinants, and so there is much less that can be determined before experimental measurements have been made.

ANALYSIS OF DRYING DURING THE FILLING PHASE

This preliminary analysis was undertaken to provide some theoretical guidance as to how to proceed with an experimental investigation. Consequently it uses the simplest possible one-dimensional model of the solids, and a fully mixed model of the gas headspace.



Notation:

- Q : Ventilation air flow rate (m^3/s)
- c : moisture concentration (kg/m^3)
- c_a : moisture concentration in atmosphere (kg/m^3)
- c_f : moisture concentration in fresh solids (kg/m^3)
- c_h : moisture concentration in headspace (kg/m^3)
- D : moisture diffusion coefficient in solids (m^2/s)
- H : mass transfer coefficient at gas-solid interface (m/s)
- k : moisture equilibrium constant between gas and solids (-)
- v : rate of filling (m/s)
- z : depth coordinate (m)

The interface level is assumed to be rising at a constant rate v . The equations are developed in a moving coordinate system, in which the rising interface remains at $z = 0$, so that the bottom of the pit corresponds to $z = vt$

Moisture is assumed to diffuse homogeneously through the solids, and evaporate at the interface. The headspace is assumed to be fully mixed.

The partial differential equation describing the moisture transport within the bed is derived from a balance over a differential slice located between z and $z+dz$

Moisture in = moisture out + moisture accumulating

$$\left[v \cdot c - D \frac{\partial c}{\partial z} \right]_z = \left[v \cdot c - D \frac{\partial c}{\partial z} \right]_{z+dz} + \frac{\partial c}{\partial t} \cdot dz$$

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial z^2} - v \cdot \frac{\partial c}{\partial z} \quad \dots\dots\dots[1]$$

The boundary and initial conditions for this partial differential equation are as follows:

$$\frac{\partial c}{\partial z} = 0 \quad \text{at} \quad z = vt \quad ; \quad t > 0 \quad (\text{No diffusion at the bottom of the pit}) \quad \dots\dots\dots[2]$$

At the surface the balance includes moisture added with fresh solids, moisture carried by the filling velocity, moisture lost by transfer to the air and moisture diffusing into or out of the underlying solids.

$$v \cdot c_f - v \cdot c - H \cdot (c - k \cdot c_h) + D \frac{\partial c}{\partial z} = 0$$

$$D \frac{\partial c}{\partial z} - (v + H) \cdot c + (Hk \cdot c_h + v \cdot c_f) = 0 \quad \text{at} \quad z = 0; \quad t > 0 \quad \dots\dots\dots[3]$$

The initial condition involves the first layer of solids with no underlying layers:

$$v \cdot c_f - v \cdot c - H \cdot (c - k \cdot c_h) = 0 \quad \text{at} \quad t = 0, \quad \text{which corresponds to } z = 0$$

(A balance between the drying rate and the addition of moisture in fresh solids)

This is rearranged to $c = \frac{v \cdot c_f + Hk \cdot c_h}{v + H} = c_0$ at $t = 0$; $z = 0$ [4]

The solution to equation [1] with boundary conditions [2] and [3] and initial condition [4] is simply

$$c = \frac{v \cdot c_f + Hk \cdot c_h}{v + H} = c_0 \quad \text{for } t > 0 \text{ and } 0 < z < vt$$

The implication of this is that the solids dry from c_f to c_0 while they remain on the surface, but once they become covered they do not dry any further until filling stops (since they are continuously being overlaid by solids with the same moisture content).

This implies that the evaporation rate is constant, and therefore c_h is constant. The evaporation rate is given by $AH \cdot (c - k \cdot c_h)$ where A is the area of the interface between solids and air.

The moisture balance for the head-space is:

$$Q \cdot c_a + AH \cdot (c - k \cdot c_h) = Q \cdot c_h \quad \text{.....[5]}$$

Eliminating c_h between [4] and [5] gives:

$$c = c_0 = \frac{(Q + AHk)v \cdot c_f + QHk \cdot c_h}{(Q + AHk)v + QH}$$

This solution can be expressed in dimensionless form:

$$\frac{(c - k \cdot c_a)}{(c_f - k \cdot c_a)} = \frac{1 + \Theta}{1 + \Theta + \Lambda} \quad \text{where } \Lambda = \frac{H}{v} \quad \text{and} \quad \Theta = \frac{AHk}{Q}$$

or alternatively

$$\frac{(c - k \cdot c_a)}{(c_f - k \cdot c_a)} = \frac{1 + \Theta}{1 + \Theta \cdot (1 + \Psi)} \quad \text{where } \Psi = \frac{\Lambda}{\Theta} = \frac{\left(\frac{Q}{A}\right)}{kv}$$

SENSITIVITY ANALYSIS

Not all the variables in these groups are readily amenable to independent control. To achieve low moisture content, Ψ should have a high value. Θ has less of an effect because it appears in both the numerator and the denominator.

The convective mass transfer coefficient H is quite strongly related to $\frac{Q}{A}$, the specific aeration rate. A rough correlation, which is suggested for situations where no better information is available, is given in the Chemical Engineer's Handbook (Perry and Green, 1999):

$h_c = 8.8 G^{0.8} / d^{0.2}$ where h_c is the heat transfer coefficient (J/m²/s/K), G is the gas mass-velocity (kg/m²/s) and d is a characteristic dimension (m) of the apparatus.

For the air-water system h_c and H are approximately related by:

$H = h_c / C_s$ where C_s the humid heat capacity of the air (J/kg/K)

$G = \rho \cdot (Q / A)$ where ρ is the density of the air (kg/m³)

$$\text{Thus } \Theta = 8.8 \frac{k \rho^{0.8}}{d^{0.2}} \left(\frac{Q}{A} \right)^{-0.2}$$

(the factor 8.8 is only correct if all quantities are expressed in SI units)

All this implies that there is very little that can be done to affect the value of Θ , since increasing Q will increase H , and these will largely compensate for each other. The aeration rate Q will be determined by details of the physical design, chiefly the stack configuration, as well as environmental factors such as wind speed. The filling rate v is the volumetric rate of solids addition divided by the surface area A , and is therefore a design parameter.

The moisture equilibrium constant k is a property of the solids and a function of temperature. In a laboratory drying experiment, samples taken from a latrine were exposed to controlled humidities between 60 and 85 % at 25 °C to determine k . Because the users had added sand to the faecal matter in the latrine, the samples were ashed to determine the organic content, which was found to be about 6 % by mass (10% by volume). Based on the assumption that all the moisture retention would be associated with the organic material in the samples, and none with the sand, the value of k at 25 °C was calculated to be $4.35 \times 10^4 \cdot x_o$, where x_o is the volume fraction of organic material in the sample.

Adding sand as a filler also has some self-compensating influences. Assuming that it is dry to start with, it immediately reduces the effective value of the initial moisture content c_f , however it also increases values of the equilibrium constant k and the filling rate v , both of which adversely affect the drying rate. In terms of the net effect on removing moisture from the system, the effect is negative, however in terms of the average dryness of the pit contents, the effect is positive. A further negative effect is that the pit will be filled substantially quicker.

This mathematical analysis has still to be backed up by a proper experimental investigation, however some preliminary observations on working latrines suggest that its main predictions are at least qualitatively correct. Samples taken from different depths showed no evidence of drying below the surface. If anything, the material appeared to get wetter with increasing depth. If this was the case, it may have been due to gravitation effects, which were not considered in the model. In this regard, the large proportion of sand found in the particular samples (± 94 % by mass) was significant, since gravitational effects are most marked in the drying of granular materials where the water does not significantly penetrate into the grains.

CONCLUSIONS

A simple model has been developed to describe the drying of faecal material in a pit during the filling phase. It shows that drying only occurs before material is covered, which results in a uniform moisture content throughout the mass. The model still requires experimental validation and calibration, but preliminary observations support its main features. The model will be used to guide the experimental programme and to interpret experimental results. To complete the picture, a model of drying in the idle phase will also need to be developed; this

will probably be more complex and rely more on experimental measurements to determine its characteristics than the model for the filling phase.

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